Pulse Power Compression by Cutting a Dense Z-Pinch with a Laser Beam

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A thin cut made through a z-pinch by an intense laser beam can become a magnetically insulated diode crossed by an intense ion beam. For larger cuts, the gap is crossed by an intense relativistic electron beam, stopped by magnetic bremsstrahlung resulting in a pointlike intense x-ray source. In either case, the impedance of the pinch discharge is increased, with the power delivered rising in the same proportion. A magnetically insulated cut is advantageous for three reasons: First, with the ion current comparable to the Alfvèn ion current, the pinch instabilities are reduced. Second, with the energy deposited into fast ions, a non-Maxwellian velocity distribution is established increasing the $\langle \sigma v \rangle$ value for nuclear fusion reactions taking place in the pinch discharge. Third, in a high density z-pinch plasma, the intense ion beam can launch a thermonuclear detonation wave propagating along the pinch discharge channel. For larger cuts the soft x-rays produced by magnetic bremsstrahlung can be used to drive a thermonuclear hohlraum target. Finally, the proposed pulse power compression scheme permits to use a cheap low power d.c. source charging a magnetic storage coil delivering the magnetically stored energy to the pinch discharge load by an exploding wire opening switch.

1. Introduction

The linear pinch configuration, the oldest thermonuclear fusion concept has been more recently revived by the availability of fast rising high voltage pulse power sources, where the pinch instabilities have less time to grow, and it is hoped that a useful amount of thermonuclear energy can be released by superfast, high density micropinches. To reach high densities by radiative collapse, the pinch current must be above the Pease-Braginskii current, and to make it more stable an axial shear flow can be superimposed on it. Here we will show that by cutting the discharge channel with an intense laser beam, the impedance of the channel can be greatly increased, and with it the power delivered to the pinch. In addition, it will be shown that such a configuration appears to be ideally suited to drive the pinch with an inductive energy storage system.

2. Making the Cut

It was shown by Tabak et al. [1] that a high intensity laser beam can bore a small hole into a plasma of supercritical density. It is here proposed to apply this technique to cut a pinch.

The required laser light intensity of wavelength λ needed to make a cut is

$$I_L(\lambda) \approx 10^{11}\ \lambda^{-2}\ \mathrm{W/cm^2}, \quad (1)$$

provided the electron number density n of the plasma is larger than the critical density

$$n_c \approx 10^{13}\ \lambda^{-2}\ \mathrm{cm^{-3}}. \quad (2)$$

For yellow light one has $\lambda = 6\times10^{-5}$ cm, and $n_c \approx 3\times 10^{21}$ cm^{-3}, and hence $I_L \sim 3\times10^{19}$ W/cm^2, feasible with petawatt laser technology [2].

The laser energy needed to make a cut through a pinch discharge is of the order

$$\varepsilon \sim I_L\, r\, d\, \tau \quad (3)$$

where r is the pinch radius, d the width of the cut, and τ the time needed to make the cut and keep it open.

With regard to laser light the cut acts like a slit of width d and length $\ell \sim r$. The larger one of d or ℓ determines the largest wave length which can pass through the cut, provided the electric vector of the laser light is perpendicular to the smaller one of d or r [3]. If $d < \lambda$, (3) has to be replaced on the r.h.s. by

$$\varepsilon \sim I_L\, r\, \lambda\, \tau \propto \text{const.}\ r\, \tau/\lambda. \quad (3a)$$

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3. Making a Magnetically Insulated Cut

The concept of magnetic insulation [4] applied to the cut goes as follows: If V is the voltage across the cut, where e and m are the charge and mass of an electron, v the electron velocity, and $\gamma=(1-v^2/c^2)^{-1/2}$, the relativistic energy equation is

$$\frac{eV}{mc^2}=\gamma-1. \tag{4}$$

Magnetic insulation then requires that the width d of the cut is larger than the electron Larmor radius

$$r_L=\frac{\gamma mvc}{eH}=\frac{mc^2}{eH}\sqrt{\gamma^2-1}<d. \tag{5}$$

By eliminating γ from (4) and (5) one obtains

$$\left(\frac{eHd}{mc^2}\right)^2>\left(1+\frac{eV}{mc^2}\right)^2-1. \tag{6}$$

With the magnetic field of the pinch (in Gaussian units) given by

$$H=\frac{2I}{rc} \tag{7}$$

one obtains from (6)

$$4\left(\frac{I}{I_A^e}\right)^2\left(\frac{d}{r}\right)^2>\left(1+\frac{eV}{mc^2}\right)^2-1, \tag{8}$$

where $I_A^e=mc^3/e=17\,000$ A is the electron Alfvèn current. The ion current across the cut is given by the Child-Langmuir law where M is the ion mass:

$$I=\frac{\sqrt{2}}{9}\left(\frac{e}{M}\right)^{1/2}\left(\frac{r}{d}\right)^2V^{3/2}. \tag{9}$$

From (8) and (9) one obtains

$$\frac{I}{I_A^e}\geq\frac{9}{4\sqrt{2}}\left(\frac{M}{m}\right)^{1/2}\left[\left(1+\frac{eV}{mc^2}\right)^2-1\right]\left(\frac{eV}{mc^2}\right)^{-3/2}, \tag{10}$$

$$\frac{d}{r}=\frac{2\sqrt{2}}{9}\left(\frac{m}{M}\right)^{1/2}\left[\left(1+\frac{eV}{mc^2}\right)^2-1\right]^{-1/2}\left(\frac{eV}{mc^2}\right)^{3/2}. \tag{11}$$

With the magnetic field decreasing towards the axis of the pinch discharge channel the magnetic insulation is imperfect, with some electron current crossing the cut. For this reason the estimates made are too optimistic, and more detailed calculations are needed.

The laser can make the cut in less than 10^{-10} s, short enough to lead to the formation of a magnetically insulated cut possessing the parameters given by (10) and (11).

Instead of (10) one can also write

$$I\geq\frac{9}{4\sqrt{2}}\sqrt{I_A^e I_A^i}\left[\left(1+\frac{eV}{mc^2}\right)^2-1\right]\left(\frac{eV}{mc^2}\right)^{-3/2}, \tag{12}$$

where $I_A^i=Mc^3/e$ is the ion Alfvèn current. For protons it is $I_A^i\approx3.1\times10^7$ A, about two thousand times larger than the electron Alfvèn current. As a function of eV/mc^2, (12) has a minimum for $eV/mc^2=2$, where $I\geq(9/2)\sqrt{I_A^e I_A^i}$, for protons $\approx3.4\times10^6$ A. From the Bennett relation $I^2=400$ NkT (I in Ampere, and $kT\sim10^{-8}$ erg for $T\sim10^8$ K), one finds for $I\sim3\times10^6$ A that $N\sim2\times10^{18}$ cm^{-1}. To make $n_c=3\times10^{21}$ cm^{-3}, requires that $r\leq10^{-2}$ cm.

According to (12) one has

$$I_A^e<I<I_A^i, \tag{13}$$

where $I\sim\sqrt{I_A^e I_A^i}$. For $I\gg I_A^e$ the pinch is unstable or "soft", realized if the current is carried by electrons, but it is stable or "stiff" if $I\ll I_A^i$, if the current is carried by ions. Since this condition can not completely be satisfied, the instabilities are only reduced, not eliminated.

For the minimum current at $eV/mc^2=2$, one obtains from (11) $d/r=(2\sqrt{2}/9)(m/M)^{1/2}$, for protons $d/r\sim7\times10^{-3}$. If the current is given in Gaussian units one must have $\pi r^2\leq I/nec$, where n is the number density of the electrons. For $I\sim3\times10^6$ A $\sim10^{16}$ esu and $n\sim10^{21}$ cm^{-3} one finds that $r\geq10^{-3}$ cm, and $d\sim10^{-5}$ cm.

Because of the smallness of the cut it is suggested to go to higher voltages where the cut becomes larger.

In the limit $eV/mc^2\gg1$ one has

$$I\geq(9/4\sqrt{2})\sqrt{I_A^e I_A^i}\,(eV/mc^2)^{1/2}, \tag{14}$$

$$d/r=(2\sqrt{2}/9)(m/M)^{1/2}(eV/mc^2)^{1/2}. \tag{15}$$

In practical units, where $I_A^e=1.7\times10^4$ A and $mc^2\approx0.5\times10^6$ eV, one has

$$I\geq3.8\times10^1\,(M/m)^{1/2}\,V^{1/2}. \tag{16}$$

For protons this is

$$I\geq1.6\times10^3\,V^{1/2} \tag{17}$$

or

$$V\leq4\times10^{-7}\,I^2. \tag{18}$$

The gap impedance is given by

$$Z=V/I=7.5\times10^{-4}\,V^{1/2}=4\times10^{-7}\,I. \tag{19}$$

In practical units, (15) is

$$d/r = 4.5\times 10^{-4}\,(m/M)^{1/2}\,V^{1/2} \tag{20}$$

for protons

$$d/r \approx 10^{-5}\,V^{1/2}. \tag{21}$$

For the example $V = 3\times 10^6$ V, according to (17) above the Pease-Braginskii current one has $d/r \sim 0.02$. By comparison, if $V = 10^8$ V, one would have $d/r = 0.1$. For a pinch radius as small as $r \sim 10^{-2}$ cm, one would have in the first and second example $d \sim 2\times 10^{-4}$ cm, and $d \sim 10^{-3}$ cm.

For $r > d$, the largest laser light wave length should be $\lambda \sim r \sim 10^{-3}$ cm with a polarization of the electric field vector in the direction of the cut, and one would have to use laser light with a wavelength distribution ranging from the infrared with $\lambda \sim 10^{-3}$ cm down to the ultraviolet with $\lambda \sim 10^{-5}$ cm. This would cover the plasma density range from $n_c^{min} \sim 10^{19}$ cm^{-3} to $n_c^{max} \approx 10^{23}$ cm^{-3}. The intensity distribution is given by (1), but according to (3a) with an energy distribution for $d < \lambda$ going in proportion of $1/\lambda$. The residual current carried across the gap is then by order of magnitude equal to

$$I \sim n_c^{min}\, r^2\, ec \approx 10\, r^2 n \text{ esu} \approx 3\times 10^{-9}\, nr^2 \text{ A}, \tag{22}$$

for the given example about $\sim 3\times 10^4$ A, well below megampere pinch currents.

The large axial electric field E_z, set up in the cut, leads to a large inward radially directed Poynting vector $\mathbf{S} = (c/4\pi)\, E_z H_\phi$, where H_ϕ is the magnetic field in the space surrounding the pinch.

The range of the fast ions crossing the cut for protons in given by [5]

$$\lambda_i = \frac{3}{8\sqrt{\pi}\,\log \Lambda}\,\frac{(kT)^{3/2}}{e^4 n}\left(\frac{M}{m}\right)^{1/2}\sqrt{E_{ion}}, \tag{23}$$

where E_{ion} is the proton energy, $\log \Lambda \sim 10$ the Coulomb logarithm, and n the plasma particle number density. Expressing E_{ion} in eV one finds

$$\lambda_i = 3.5\times 10^7\,(T^{3/2}/n)\sqrt{E_{ion}}. \tag{24}$$

For the example $T = 10^8$ K, $n = 5\times 10^{22}$ cm^{-3} (corresponding solid state density), one finds for $E_{ion} = 3\times 10^6$ eV that $\lambda_i \sim 1$ cm, and for $E_{ion} = 10^8$ eV that $\lambda_i \sim 7$ cm. For deuterons the range is twice as large. As expected, the high voltage leads to a larger range, and hence to a larger length over which the stability of the pinch is increased. Over this same length there is a departure from a Maxwellian velocity distribution by a fast ion component. This, of course, increases the $\langle \sigma v \rangle$ value for a fusion reaction, possibly sufficient to reach burn for the neutronless HB^{11} reaction.

Under radiative collapse of the pinch discharge channel the highest densities are reached if the plasma temperature is as low as possible. But even then, the cut leads to a burst of energetic ions. If, for example, $T \sim 10^6$ K and $n \sim 5\times 10^{23}$ cm^{-3} (corresponding to 10 times solid density), one finds that now $\lambda_i \sim 10^{-3}$ cm, which means that a hot spot is created at the location of the cut, from which a thermonuclear detonation wave may be launched propagating along the pinch channel [6].

4. Pulse Power Compression by Making the Cut

For the example $I = 3\times 10^6$ A, one obtains from (18) that $V \sim 3\times 10^6$ V, and from (19) that $Z \sim 1\ \Omega$. But for $V = 10^8$ V one would have $I \sim 1.67\times 10^7$ A and $Z = 6.4\ \Omega$. By comparison, a 10 cm long plasma column with a cross section $\sim 10^{-4}$ cm^2 and a temperature of $\sim 10^8$ K would have a resistance equal to $R \approx 6\times 10^{-3}\ \Omega$, with the resistive losses about 10^3 times smaller. Therefore, a magnetically insulated cut increases the power dissipated into the pinch $\sim 10^3$ fold, leading to a large pulse power compression. With the pinch discharge as the load, the cut acts as a fast switch dissipating at the location of the switch the magnetically stored energy into the load.

The magnetic energy stored in the space surrounding the pinch of length ℓ and return current conductor radius R is

$$\varepsilon_M = 10^{-9}\, I^2\, \ell\, \log(R/r) \text{ Joule}. \tag{25}$$

For the example for $I = 3\times 10^6$ A, $\ell = R = 10$ cm, $r \sim 10^{-2}$ cm, one has $\varepsilon_M \sim 1$ MJ, and for $I = 1.6\times 10^7$ A, $\varepsilon_M \sim 30$ MJ.

The inductance of the discharge channel is

$$L = 2\times 10^{-9}\, \ell\, \log(R/r) \text{ Henry} \tag{26}$$

for the given example $\ell \sim R = 10$ cm, $r \sim 10^{-2}$ cm equal to $L \sim 10^{-7}$ Henry.

The discharge time is

$$\tau = L/Z \text{ sec}. \tag{27}$$

For $Z \approx 1\ \Omega$, where $I \approx 3\times 10^6$ A, $V \sim 3\times 10^6$ V, one has $\tau \sim 10^{-7}$ sec, and for $Z \approx 6\ \Omega$, where $I \sim 1.7\times 10^7$ A, $V \sim 10^8$ V, one has $\tau \sim 2\times 10^{-8}$ sec.

A pinch radius $r \sim 10^{-2}$ cm, a cut with a width $d \sim 10^{-3}$ cm, and a time $\tau \sim 10^{-8}$ sec needed to keep the cut open, would according to (1) and (3) require the laser energy $\varepsilon \sim 3\times 10^4$ J.

5. Making a Larger Cut

If the cut is made larger and violates the magnetic insulation criterion, the cut is bridged by an intense relativistic electron beam. In the limit of high relativistic electron energies, the repulsive space charge is compensated by attractive magnetic forces. As a result, the beam current remains equal to the current carried by the pinch. The voltage across the cut is given by

$$V \sim LI/\tau . \quad (28)$$

For $L \sim 10^{-7}$ Henry, $I \sim 3\times10^{6}$ A and $\tau \sim 3\times10^{-8}$ sec, one has $V \sim 10^{7}$ V.

After reentering the plasma on the other side of the cut, the electron beam can propagate inside the plasma only as long as $I \le I_A^e \beta\gamma$. For an electron energy of 3×10^{7} eV one has $\beta \approx 1$ and $\gamma \approx 20$, hence $I_A^e \beta\gamma \approx 5\times10^{5}$ A and thus $I = 3\times10^{6}$ A $> I_A^e \beta\gamma$. In this case the electrons are forced into Larmor motion around the selfmagnetic beam field, preventing the beam from propagating. Losing their energy by magnetic bremsstrahlung [7], they are brought to rest over a distance given by ($\gamma \gg 1$)

$$\lambda_e \sim \frac{(mc^2)^4}{e^4 H^2} \frac{1}{E} , \quad (29)$$

where E is the electron energy. With $E_0 = mc^2 \sim 5\times10^{5}$ eV and $e^2/r_0 = mc^2$, where r_0 is the classical electron radius, this can be written as

$$\lambda_e \sim (mc^2/r_0^2)\,(1/H^2)\,(E_0/E) . \quad (30)$$

For $\gamma \sim 20$, $(E/E_0 \approx 20)$ and $H \sim 10^{8}$ G (valid for $I = 5\times10^{6}$ A, $r = 10^{-2}$ cm), one finds $\lambda_e \sim 10^{-5}$ cm. Because of this short range, the side of the cut where the electron beam reenters the plasma becomes an intense x-ray point source with a power equal to

$$P = IV \sim LI^2/\tau . \quad (31)$$

For $L \sim 10^{-7}$ Henry, $I \sim 10^{7}$ A, $\tau \sim 10^{-8}$ s one has $P \sim 10^{14}$ Watt. The maximum of this radiation occurs at the frequency

$$\omega_{max} \approx (eH/mc)\,\gamma^2 \quad (32)$$

with the photon energy

$$E_{max} = \hbar\,\omega_{max} = e\,(\hbar/mc)\,H\gamma^2 \approx 10^{-8} H\gamma^2 \text{ eV}. \quad (33)$$

For the example $H \sim 10^{8}$ G, $\gamma \approx 20$ one finds $E_{max} \sim 400$ eV. This is a very interesting result, because it shows that the x-ray point source can be used to drive a thermonuclear hohlraum-target, with the hohlraum placed near the cut, for example on the axis of the pinch. It should be emphasized that this configuration is possible with an exploding wire pinch discharge.

6. Further Comments on Pulse Power Compression

Going to higher voltages leads to shorter discharge times and higher pulse power. This suggests to drive the discharge by inductively stored energy. There are two reasons why inductive energy storage is superior to capacitive energy storage: First, in inductive energy storage large electric fields occur only in the last moment during which the energy is dumped into the load, whereas in capacitive energy storage the large electric fields in the Marx generator must be sustained longer by several orders of magnitude. Second, in inductive storage, the maximum energy density is limited by the mechanical strength of the storage coil, whereas in capacitive energy storage it is limited by the electric breakdown field strength. Therefore, the maximum energy storage density of a magnetic field can be about three orders of magnitude larger than for an electric field, making inductive energy storage about 10^3 times more compact than capacitive energy storage. And because a magnetic field coil can be magnetized rather slowly, the charging can be done by a cheap low power d.c. source.

As it was shown by Salge and his group [8], a mechanically moved switch in combination with an exploding wire fuse can generate a voltage of ~100 kV on a time-scale of $\sim 10^{-7}$ sec. Following this idea one may consider the following scenario, illustrated in Figure 1:

1. A comparatively cheap low power d.c. power source magnetizes a coil storing an energy of several MJ. (If done on a time scale of ~0.1 sec, the d.c. power is $\sim 10^{9}$ Watt, attainable with homopolar generators, and with a d.c. voltage of ~100 V the current would have to be $\sim 10^{7}$ A).
2. A mechanically moved switch opens the coil with the current diverted to pass through a tamped wire exploding in $\sim 10^{-7}$ sec. (For a ~10 cm long wire the voltage rises there to ~100 kV).
3. The high voltage generated by the exploding wire triggers the pinch discharge subsequently cut by a laser beam further increasing the voltage and pulse power.

The initial input power can be reduced with the concept of the Xram [9], where a bank of coils is charged in series and discharged in parallel, adding the currents of all coils. There the current is equally divided between many switches. For a bank of 100 coils, for example, the

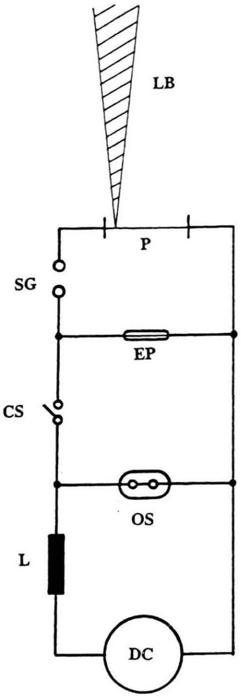

Fig. 1. DC homopolar generator, L storage coil; OS mechanical opening switch; CS mechanical closing switch; EP exploding wire opening switch; SG spark gap closing switch; P pinch discharge; LB laser beam.

current in each switch would be reduced 100-fold, for a current of $\sim 10^7$ A, down to 10^5 A.

A mechanical switch which can open a current of $\sim 10^5$ A was proposed and tested by Braunsberger et al. [10].

The pulse power compression scenario thus looks as follows:

1. A 10^7 Watt, 10^2 V, 10^5 A d.c. source magnetizes a bank of 100 coils in series.
2. By mechanically opening the switches connecting the coils and switching them in parallel, their currents are added up, increasing the pulse power to 10^9 Watt.
3. The added up current of 10^7 A passes through a tamped wire which explodes in $\sim 10^{-7}$ sec, raising the voltage from 10^2 volt to 10^5 V, and the pulse power from 10^9 Watt to 10^{12} Watt.
4. The pinch discharge ignited by the 10^{12} Watt pulse is cut by a laser beam raising the voltage from 10^5 to 10^8 V, and the power to 10^{15} Watt.

The high voltage pulse in the last step occurs on a timescale of 10^{-8} sec, short enough for the insulating material to prevent breakdown by streamer formation, while the inertia of the magnetic field energy stored in the space surrounding the pinch keeps the current constant.

We estimate the dimensions of the magnetic storage coil, required to store an energy of ~ 10 MJ to be discharged in a $\sim 3\times10^{-8}$ sec, as follows: In electrostatic units the selfinductance L of a coil with a wire of length ℓ and height h is approximately given by $L \sim \ell^2/h$, and its capacitance is approximately given by $C \approx h/\log(R/r)^2$, where r is the coil radius and R the radius of the wall "containing" the coil. One thus has $LC \sim \ell^2/\log(R/r)^2$, and for the discharge time

$$\tau_c \sim \sqrt{LC}/c = (\ell/c)/\sqrt{\log(R/r)^2}\,. \tag{34}$$

The impedance of the coil expressed in practical units is

$$Z = 30\,(\ell/h)\sqrt{\log(R/r)^2}\ \Omega \tag{35}$$

for the example $\ell \sim h$ it is about 30 Ω. For a current of 3×10^6 A, the voltage pulse would thus rise to $\sim 10^8$ V.

To store an energy of ~ 10 MJ at a magnetic field strength of $\sim 5\times10^4$ Gauss (possessing a magnetic pressure of ~ 100 atm), requires a volume of $\sim 1\ \mathrm{m}^3$. By comparison, the volume of a capacitor bank storing the same amount of energy would be of the order $10^3\ \mathrm{m}^3$. Ignoring the logarithmic factor of the order unity, the discharge time is simply given by ℓ/c, the time needed for an electromagnetic pulse propagating along the coil wire with the velocity of light. For a discharge time of $\sim 3\times10^{-8}$ sec the length of the coil wire would have to be $\ell \approx 10$ m. In the Xram circuit, this is the wire length for each coil, because the time to discharge all coils does not change if the coils are switched in parallel.

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